

Objective: to find the longitudinal gradient  $G_z$  generated by Maxwell coil pair.

Helmholtz coils and Maxwell coils are used in MRI to produce the transverse and the longitudinal gradient ( $G_{xy}$  and  $G_z$ ) respectively. Figure 1 shows the difference between the two coils. For Helmholtz pair coils we have the separation distance  $d$  is equal to that of the radius  $a$  of the loop and the current  $I$  in the two coils run in the same direction. However for Maxwell pair coils we have  $d = (3)^{1/2}a$  and the current in the two coils run in opposite direction. These will yield homogeneity in the magnetic field till the third order derivative for Helmholtz pair coils and till the fourth order for Maxwell pair coils.

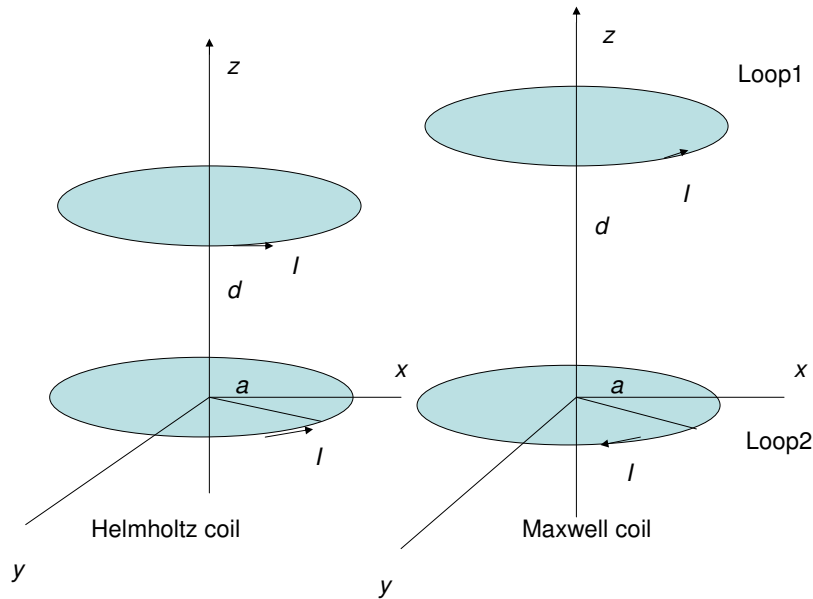


Figure 1

The Magnetic field for the Helmholtz pair coils and Maxwell pair coils on the  $z$  axis can be calculated by using Biot- Savart Law.

The magnetic field  $B$  along  $z$  axis for Helmholtz pair coil is:

$$B_z = \frac{\mu_0 I a^2}{2[(d - z)^2 + a^2]^{3/2}} + \frac{\mu_0 I a^2}{2[z^2 + a^2]^{3/2}}$$

The magnetic field  $B$  along  $z$  axis for Maxwell pair coil is:

$$B_z = \frac{\mu_0 I a^2}{2[(d - z)^2 + a^2]^{3/2}} - \frac{\mu_0 I a^2}{2[z^2 + a^2]^{3/2}}$$

Figure 2 shows the magnetic field  $B$  for the two coils for  $a = 10\text{cm}$ ,  $I = 1.5\text{ A}$ . One coil is located at  $z = 0$  and the second coil located at  $z = a$  for Helmholtz coil pair or  $z = 1.73 a$  for Maxwell coil pair. Separation unit  $z$  is in meter.

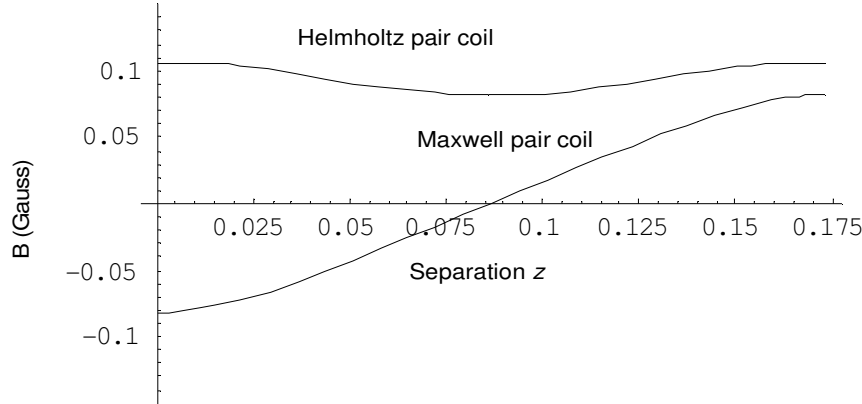


Figure 2 ( $a = 10\text{ cm}$ ,  $I = 1.5\text{A}$ )

Magnetic field off the  $z$  axis is divided into two parts  $B_z$  direction and  $B_r$  direction:

For loop 1:

$$B_z = \frac{\mu_0 I k_1}{4\pi\sqrt{ar}} \left( K_1 + \frac{a^2 - r^2 - (d-z)^2}{((a-r)^2 + (d-z)^2)} E_1 \right)$$

$$B_r = \frac{\mu_0 I k_1 (d-z)}{4\pi r \sqrt{ar}} \left( -K_1 + \frac{a^2 + r^2 + (d-z)^2}{((a-r)^2 + (d-z)^2)} E_1 \right)$$

where  $r =$  radial distance ( $r = (x^2 + y^2)^{(1/2)}$ ),  $K_1$  and  $E_1$  are elliptic integrals of the first and second kind of  $k_1$  such that:

$$k_1 = \sqrt{\frac{4ar}{(a+r)^2 + (d-z)^2}}$$

For loop2

$$B_z = \frac{\mu_0 I k_2}{4\pi\sqrt{ar}} \left( K_2 + \frac{a^2 - r^2 - z^2}{((a-r)^2 + z^2)} E_2 \right)$$

$$B_r = \frac{\mu_0 I k_2 z}{4\pi r \sqrt{ar}} \left( -K_2 + \frac{a^2 + r^2 + z^2}{((a-r)^2 + z^2)} E_2 \right)$$

Same as above,  $K_2$  and  $E_2$  are elliptic integrals of the first and second kind of  $k_2$  such that:

$$k_2 = \sqrt{\frac{4ar}{(a+r)^2 + z^2}}$$

Figure 3 and Figure 4 show the total magnetic field  $B_z$  and  $B_r$  off the  $z$  axis for Maxwell coil pairs.

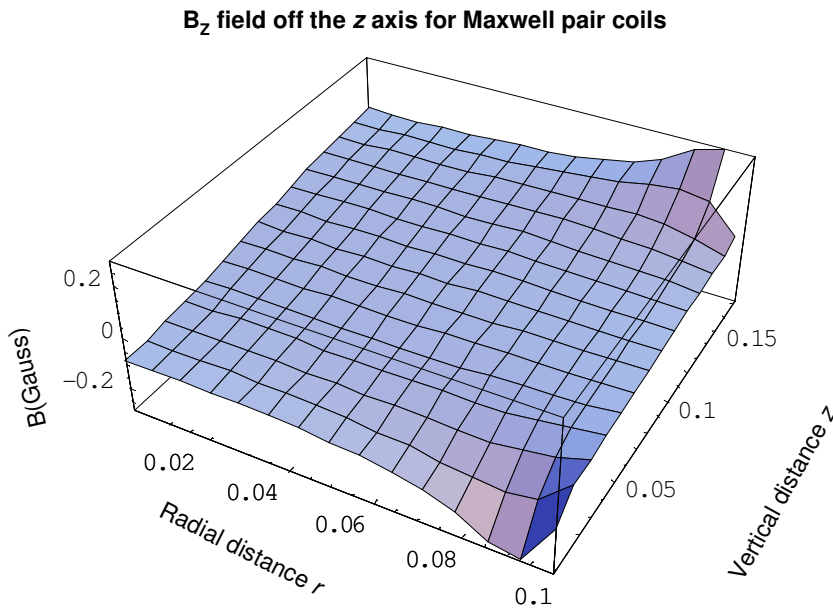


Figure 3( $a = 10$  cm,  $I = 1.5$ A)

**$B_r$  field off the  $z$  axis for Maxwell pair coils**

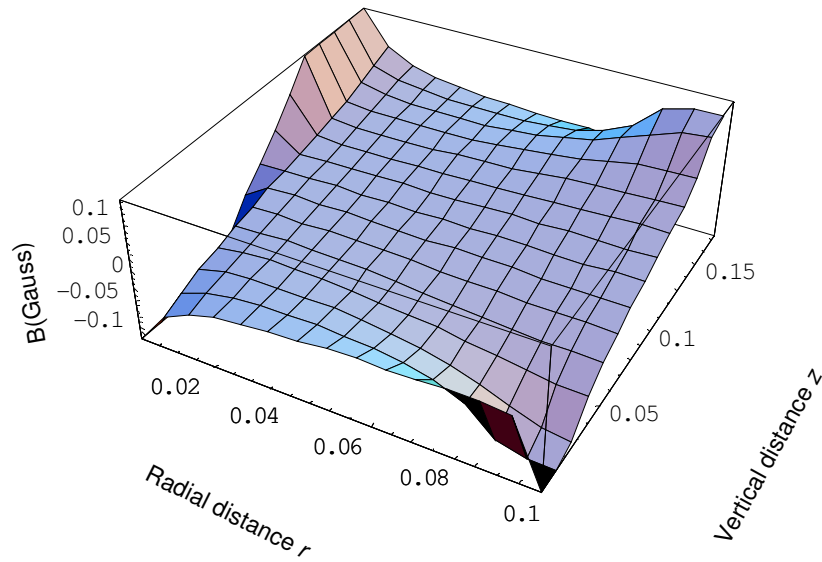


Figure 4( $a = 10$  cm,  $I = 1.5$ A)

Next I will try to plot the  $B_z$  field for particular values of  $r$  :

**$B_z$  field off the  $z$  axis for  $r = 2.5$  cm**

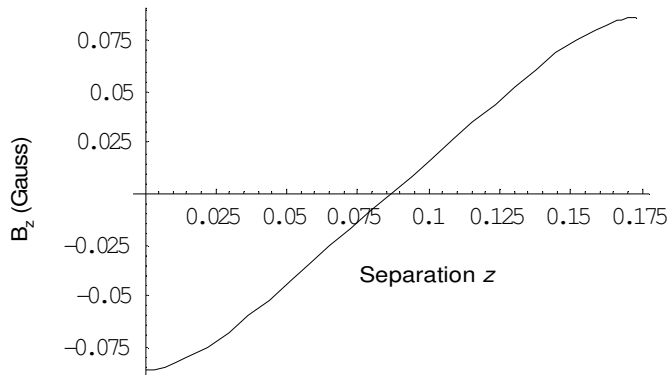


Figure 5( field off the  $z$  axis for Maxwell pair coils such that  $a = 10$  cm and  $I = 1.5$ A)

**$B_z$  field off the z axis for  $r = 5$  cm**

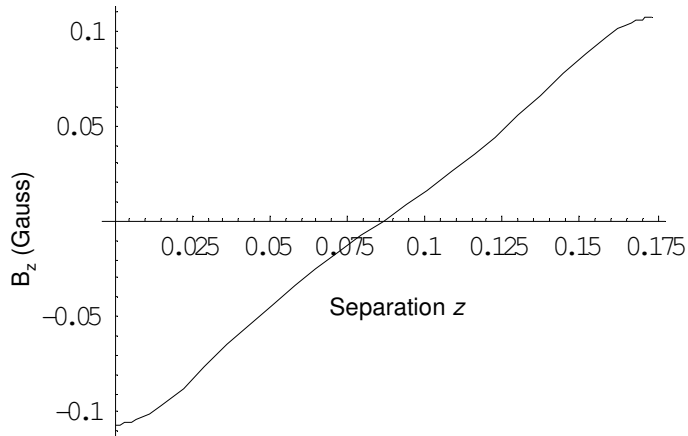


Figure 6 (field off the z axis for Maxwell pair coils such that  $a = 10$  cm and  $I = 1.5$ A)

**$B_z$  field off the z axis for  $r = 7.5$  cm**

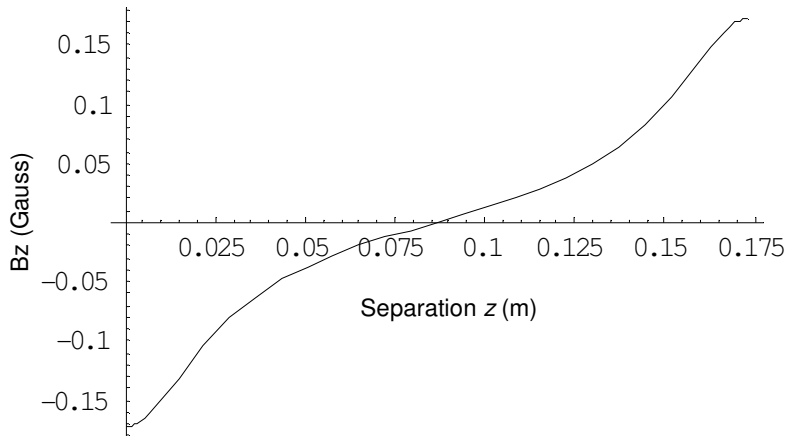


Figure 7 (field off the z axis for Maxwell pair coils such that  $a = 10$  cm and  $I = 1.5$ A)

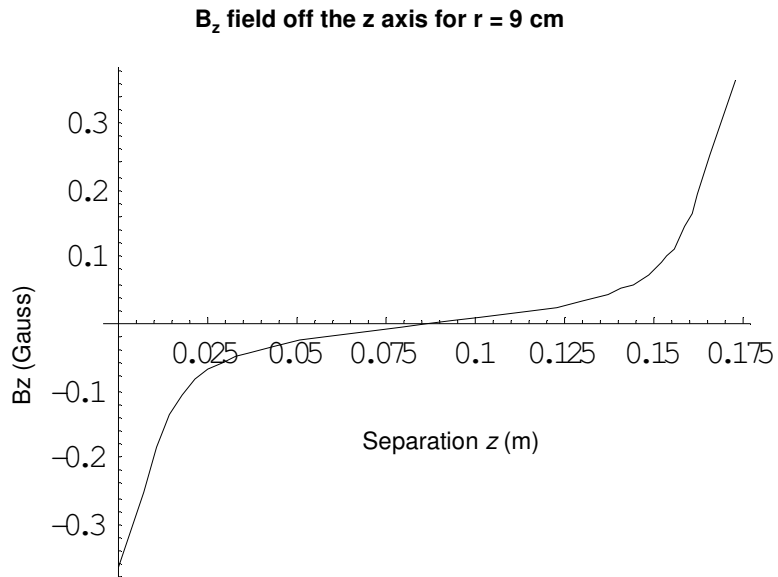


Figure 8 (field off the z axis for Maxwell pair coils such that  $a = 10$  cm and  $I = 1.5$ A)

The gradient  $G_z$  for Maxwell pair coils is  $G_z = \frac{\partial B_z}{\partial z}$

**$G_z$  field off the  $z$  axis for Maxwell pair coils**

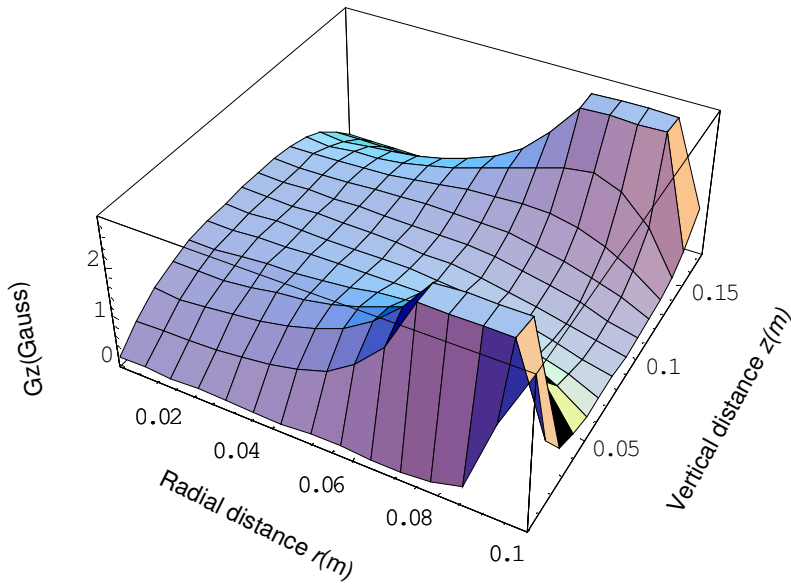


Figure 9 ( $a = 10\text{cm}$  and  $I = 1.5\text{A}$ )

I can also plot the  $B_r$  field for the Maxwell coil pair for particular value of  $r$ .

**$B_r$  field off the  $z$  axis for  $r = 2.5\text{ cm}$**

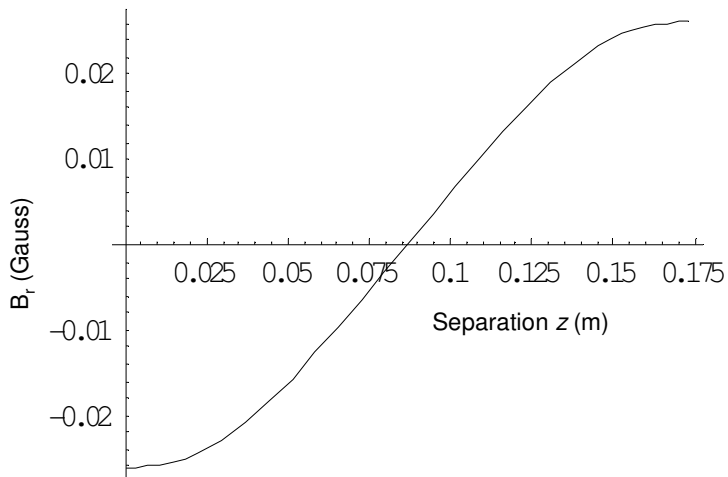


Figure 10 (field off the  $z$  axis for Maxwell pair coils such that  $a = 10\text{ cm}$  and  $I = 1.5\text{A}$ )

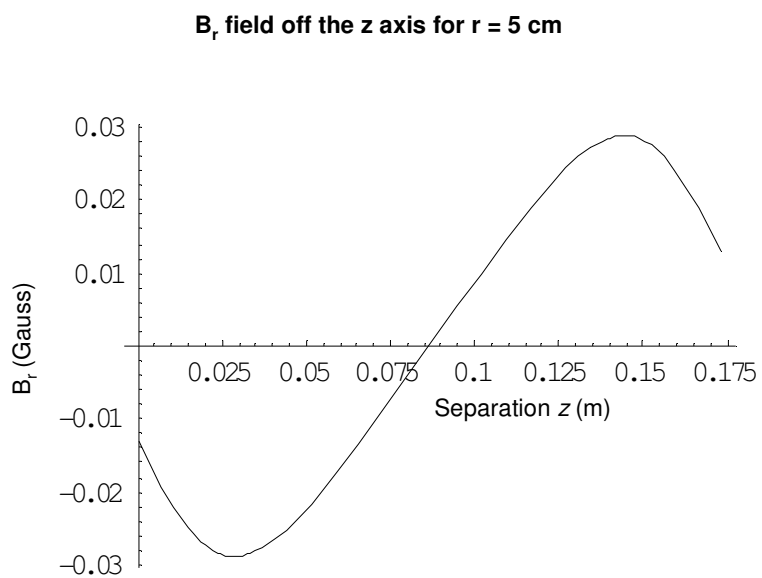


Figure 11 (field off the z axis for Maxwell pair coils such that  $a = 10$  cm and  $I = 1.5$ A)

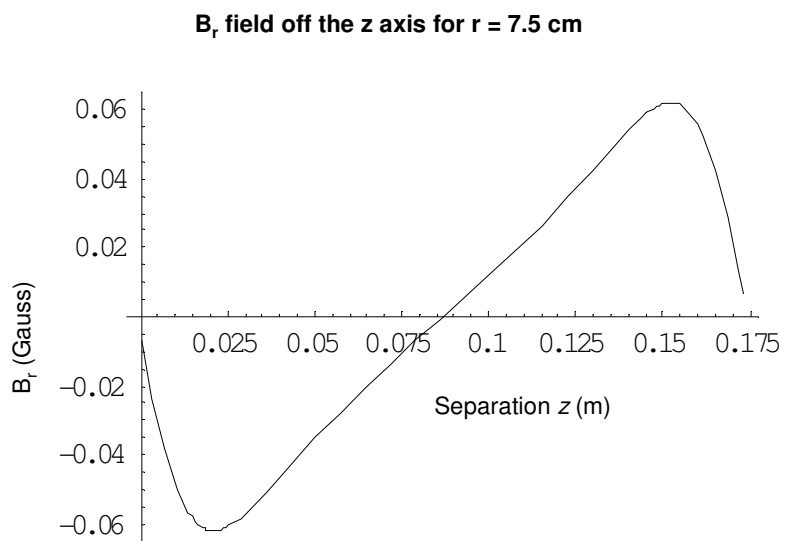


Figure 12 (field off the z axis for Maxwell pair coils such that  $a = 10$  cm and  $I = 1.5$ A)

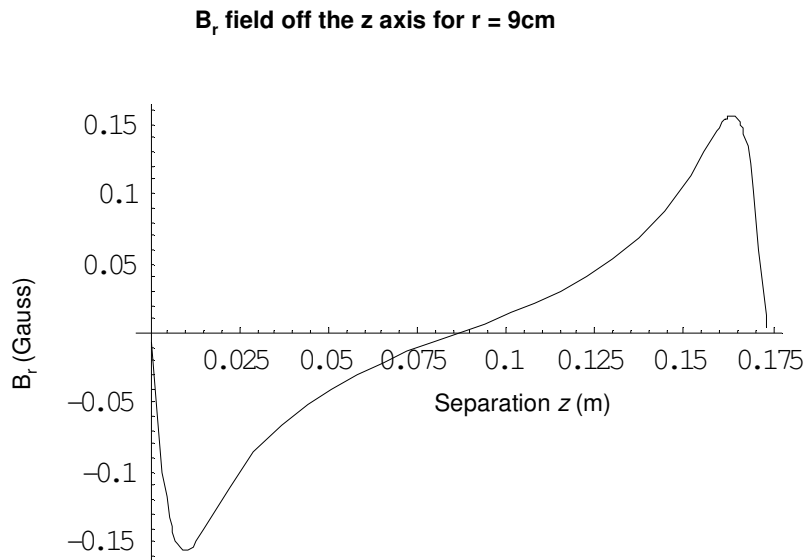


Figure 13 (field off the z axis for Maxwell pair coils such that  $a = 10\text{ cm}$  and  $I = 1.5\text{A}$ )

Figure 14 and 15 show plots for the gradient  $G_z$  for different values of current and radius. In Figure 14 where the radius is twice as much as it was before we see decrease in the gradient value compare to Figure 9 by a factor of 3. However in Figure 15 the strength of gradient increases with increasing current, double current will double the value of the gradient (that make since  $B$  is proportional to  $I$ ).

**$G_z$  field off the z axis for Maxwell pair coils**

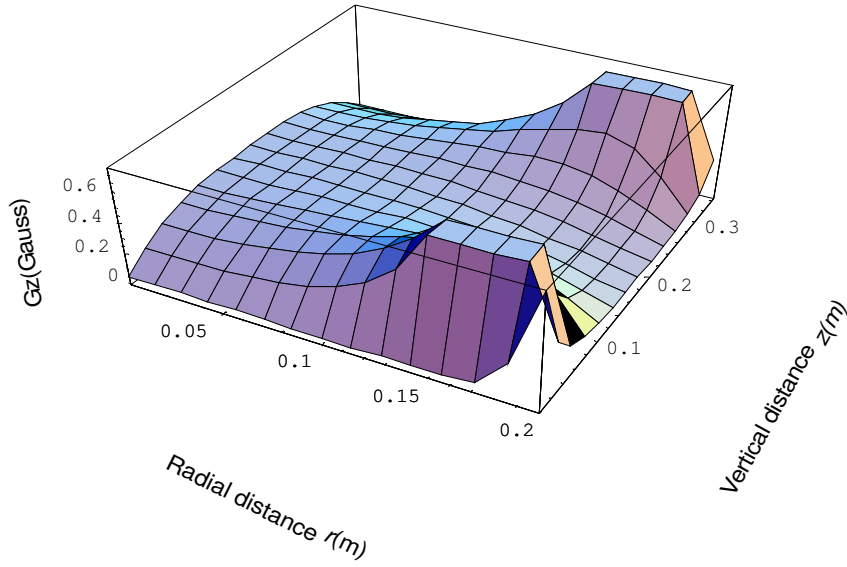


Figure 14 ( $a = 20$  cm and  $I = 1.5$  A)

**$G_z$  field off the z axis for Maxwell pair coils**

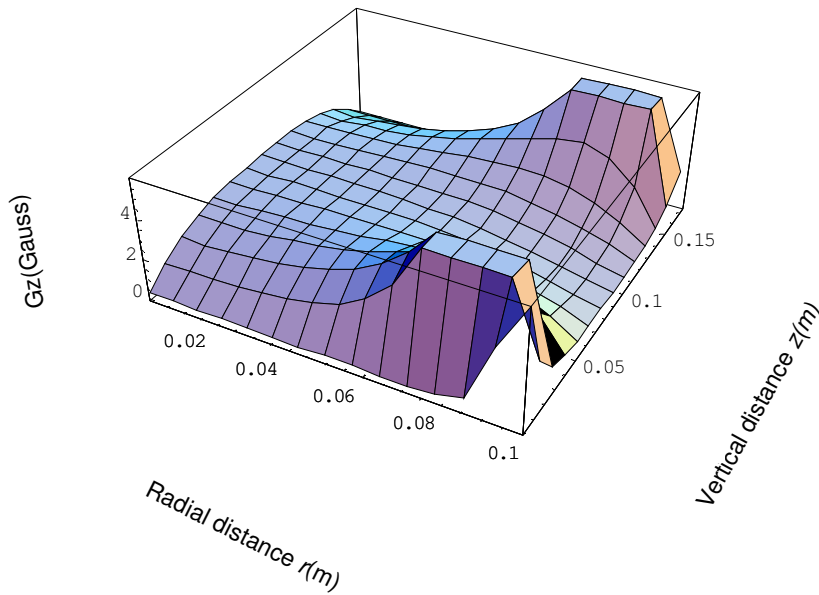


Figure 15 ( $a = 10$  cm and  $I = 3$  A)