

Objective: to find the longitudinal gradient G_z generated by Maxwell coil pair.

Helmholtz coils and Maxwell coils are used in MRI to produce the transverse and the longitudinal gradient (G_{xy} and G_z) respectively. Figure 1 shows the difference between the two coils. For Helmholtz pair coils we have the separation distance d is equal to that of the radius a of the loop and the current I in the two coils run in the same direction. However for Maxwell pair coils we have $d = (3)^{1/2}a$ and the current in the two coils run in opposite direction. These will yield homogeneity in the magnetic field till the third order derivative for Helmholtz pair coils and till the fourth order for Maxwell pair coils.

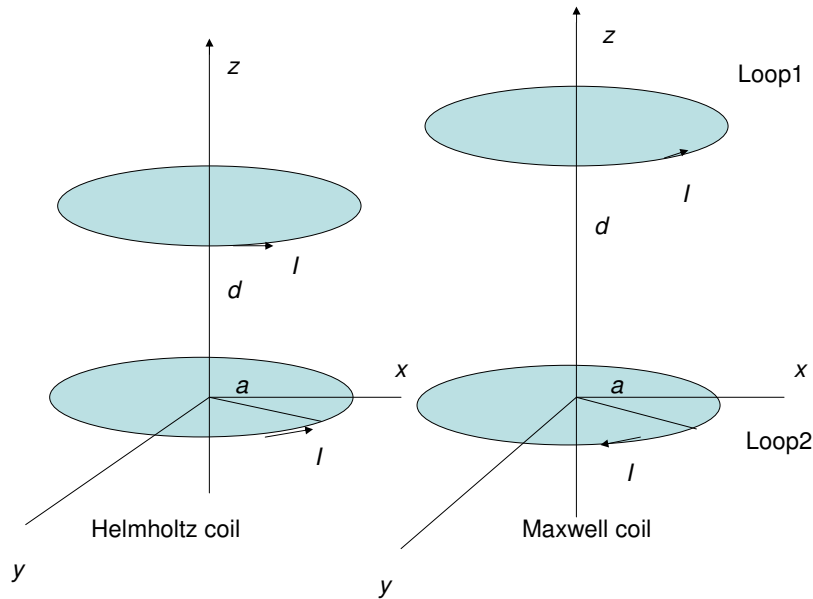


Figure 1

The Magnetic field for the Helmholtz pair coils and Maxwell pair coils on the z axis can be calculated by using Biot- Savart Law.

The magnetic field B along z axis for Helmholtz pair coil is:

$$B_z = \frac{\mu_0 I a^2}{2[(d - z)^2 + a^2]^{3/2}} + \frac{\mu_0 I a^2}{2[z^2 + a^2]^{3/2}}$$

The magnetic field B along z axis for Maxwell pair coil is:

$$B_z = \frac{\mu_0 I a^2}{2[(d - z)^2 + a^2]^{3/2}} - \frac{\mu_0 I a^2}{2[z^2 + a^2]^{3/2}}$$

Figure 2 shows the magnetic field B for the two coils for $a = 10\text{cm}$, $I = 1.5\text{ A}$. One coil is located at $z = 0$ and the second coil located at $z = a$ for Helmholtz coil pair or $z = 1.73 a$ for Maxwell coil pair. Separation unit z is in meter.

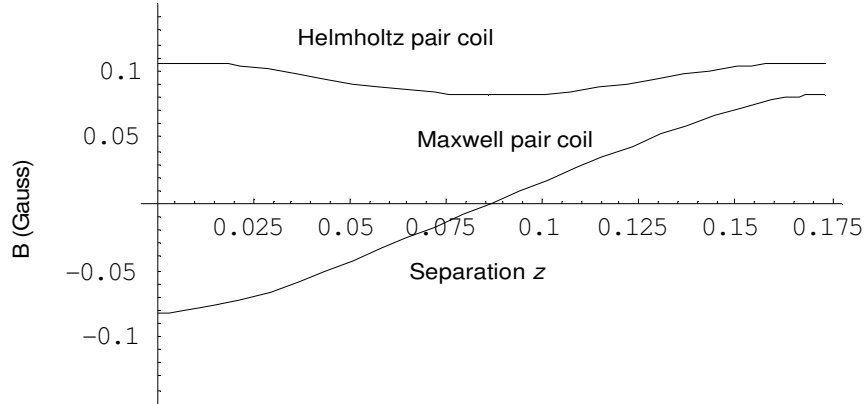


Figure 2 ($a = 10\text{ cm}$, $I = 1.5\text{A}$)

Magnetic field off the z axis is divided into two parts B_z direction and B_r direction:

For loop 1:

$$B_z = \frac{\mu_0 I k_1}{4\pi\sqrt{ar}} \left(K_1 + \frac{a^2 - r^2 - (d-z)^2}{((a-r)^2 + (d-z)^2)} E_1 \right)$$

$$B_r = \frac{\mu_0 I k_1 (d-z)}{4\pi r \sqrt{ar}} \left(-K_1 + \frac{a^2 + r^2 + (d-z)^2}{((a-r)^2 + (d-z)^2)} E_1 \right)$$

where $r =$ radial distance ($r = (x^2 + y^2)^{(1/2)}$), K_1 and E_1 are elliptic integrals of the first and second kind of k_1 such that:

$$k_1 = \sqrt{\frac{4ar}{(a+r)^2 + (d-z)^2}}$$

For loop2

$$B_z = \frac{\mu_0 I k_2}{4\pi\sqrt{ar}} \left(K_2 + \frac{a^2 - r^2 - z^2}{((a-r)^2 + z^2)} E_2 \right)$$

$$B_r = \frac{\mu_0 I k_2 z}{4\pi r \sqrt{ar}} \left(-K_2 + \frac{a^2 + r^2 + z^2}{((a-r)^2 + z^2)} E_2 \right)$$

Same as above, K_2 and E_2 are elliptic integrals of the first and second kind of k_2 such that:

$$k_2 = \sqrt{\frac{4ar}{(a+r)^2 + z^2}}$$

Figure 3 and Figure 4 show the total magnetic field B_z and B_r off the z axis for Maxwell coil pairs.

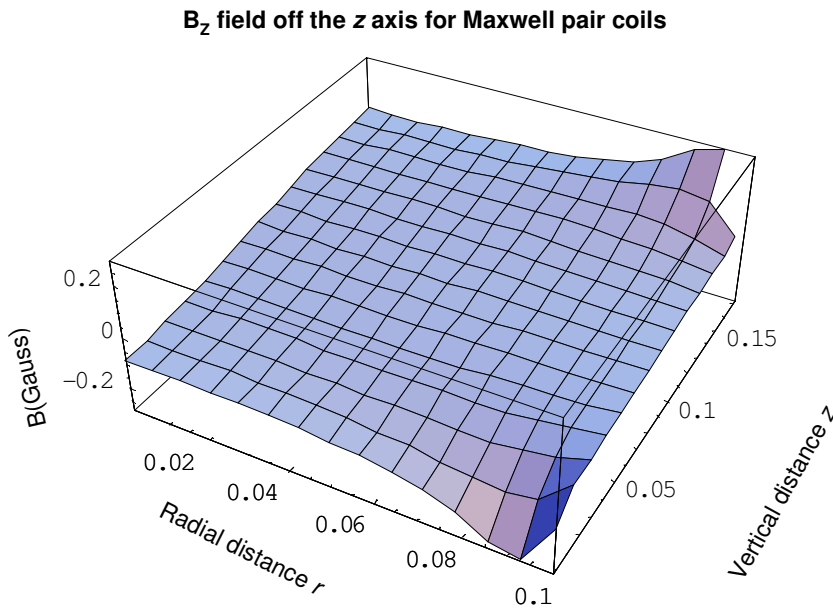


Figure 3($a = 10$ cm, $I = 1.5$ A)

B_r field off the z axis for Maxwell pair coils

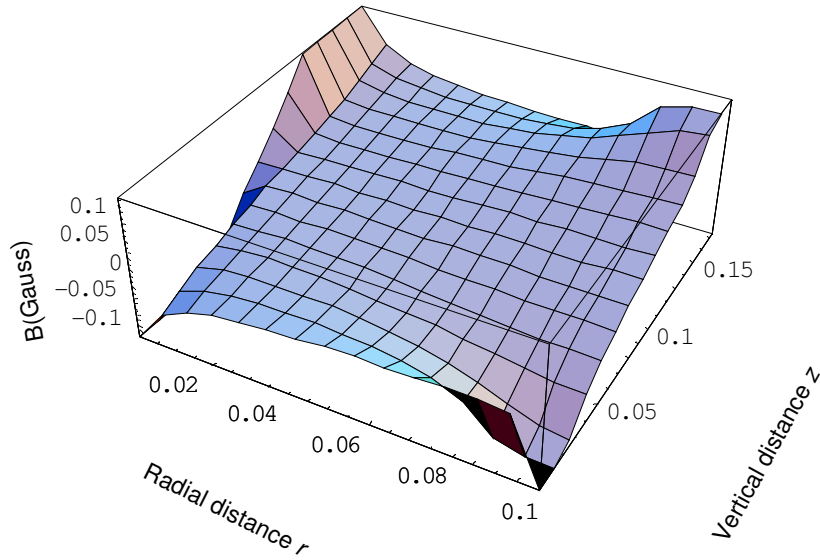


Figure 4($a = 10$ cm, $I = 1.5$ A)

Next I will try to plot the B_z field for particular values of r :

B_z field off the z axis for $r = 2.5$ cm

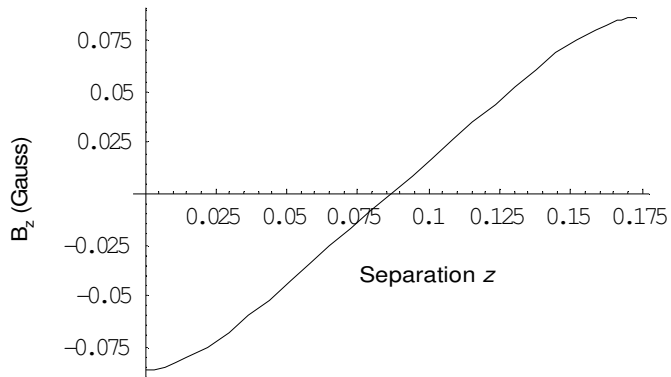


Figure 5(field off the z axis for Maxwell pair coils such that $a = 10$ cm and $I = 1.5$ A)

B_z field off the z axis for $r = 5$ cm

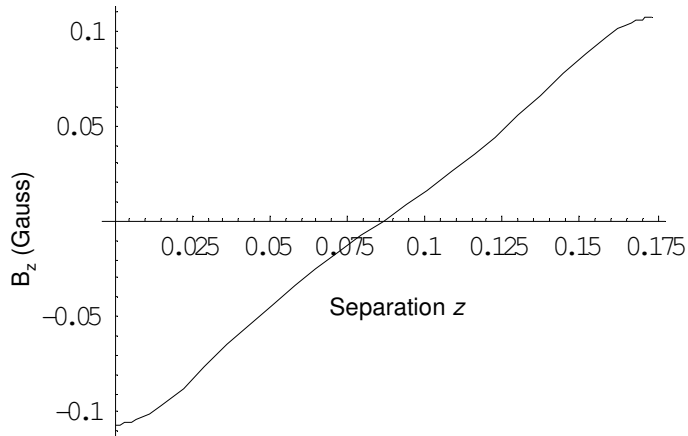


Figure 6 (field off the z axis for Maxwell pair coils such that $a = 10$ cm and $I = 1.5$ A)

B_z field off the z axis for $r = 7.5$ cm

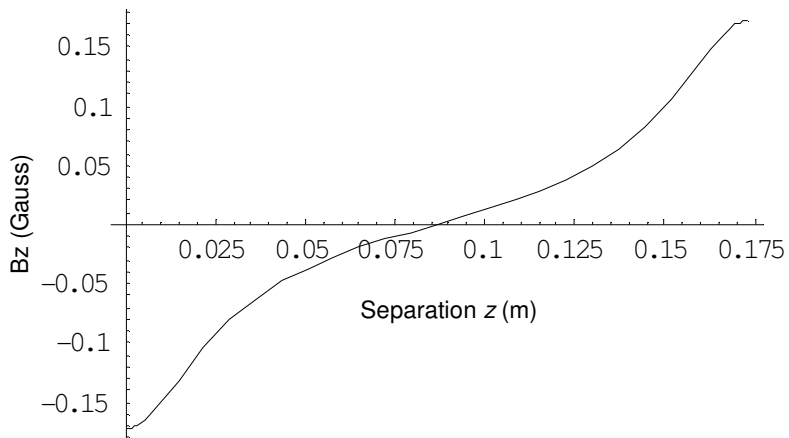


Figure 7 (field off the z axis for Maxwell pair coils such that $a = 10$ cm and $I = 1.5$ A)

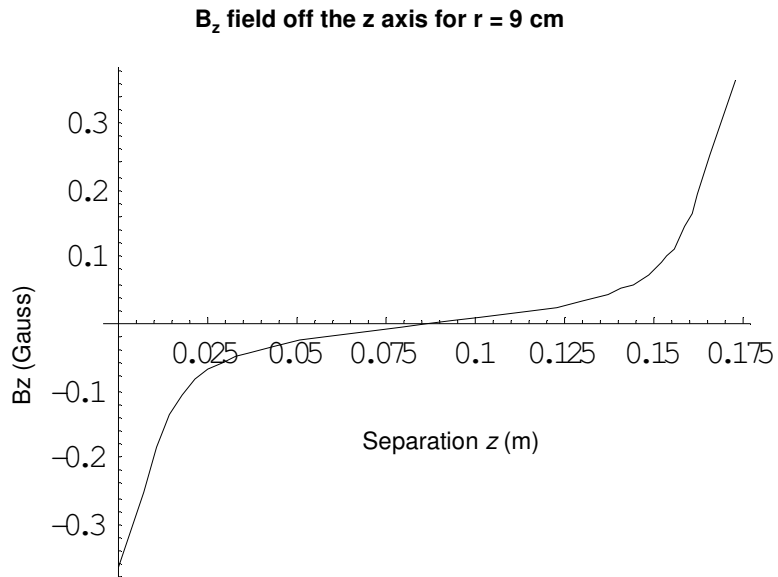


Figure 8 (field off the z axis for Maxwell pair coils such that $a = 10$ cm and $I = 1.5$ A)

The gradient G_z for Maxwell pair coils is $G_z = \frac{\partial B_z}{\partial z}$

G_z field off the z axis for Maxwell pair coils

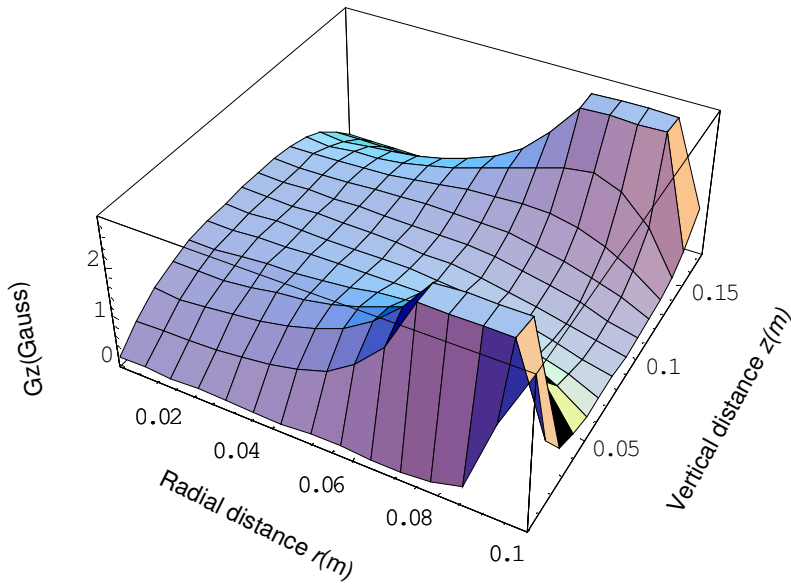


Figure 9 ($a = 10\text{cm}$ and $I = 1.5\text{A}$)

I can also plot the B_r field for the Maxwell coil pair for particular value of r .

B_r field off the z axis for $r = 2.5\text{ cm}$

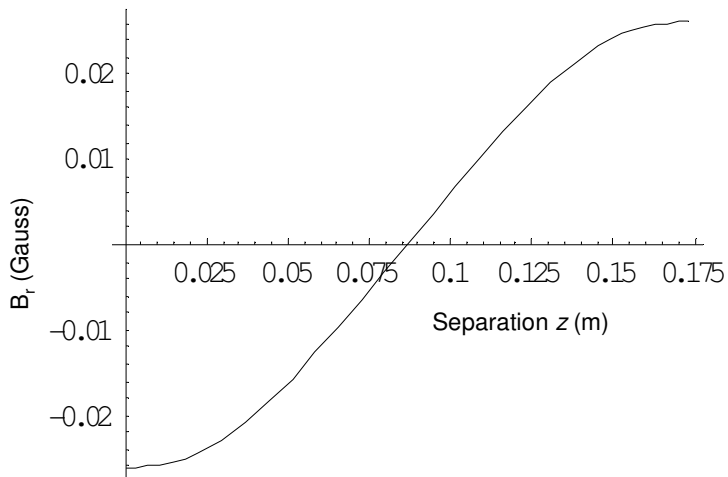


Figure 10 (field off the z axis for Maxwell pair coils such that $a = 10\text{ cm}$ and $I = 1.5\text{A}$)

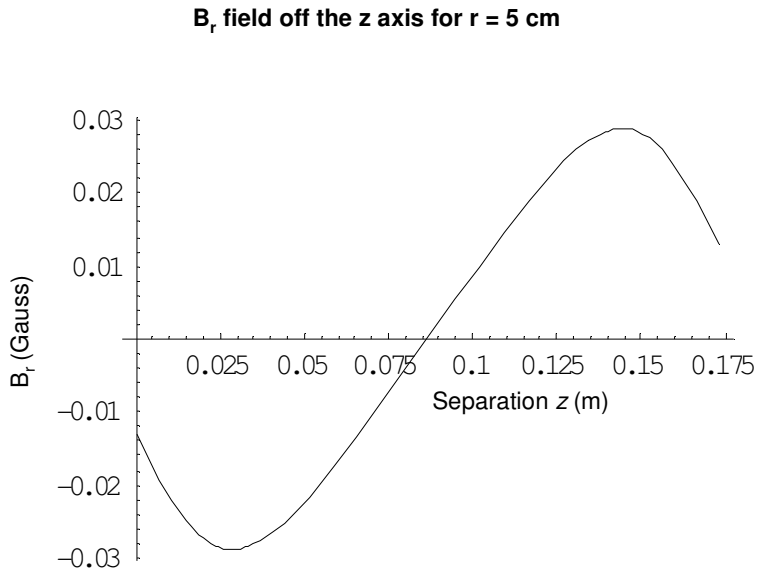


Figure 11 (field off the z axis for Maxwell pair coils such that $a = 10$ cm and $I = 1.5$ A)

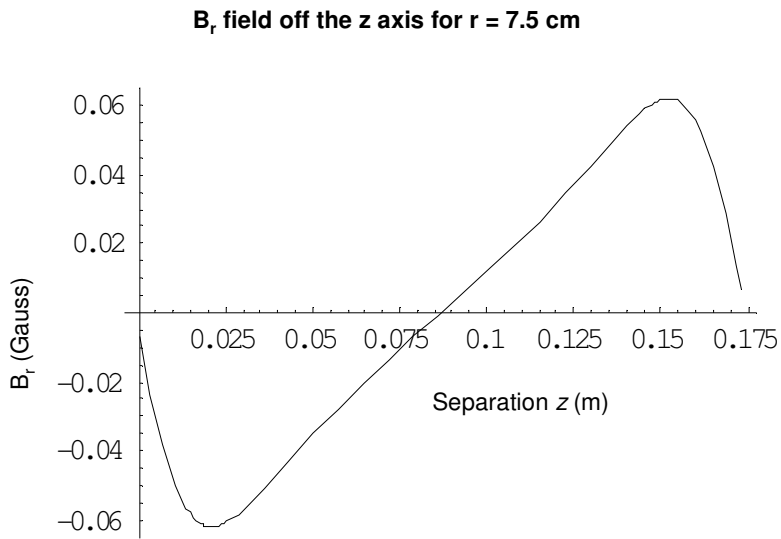


Figure 12 (field off the z axis for Maxwell pair coils such that $a = 10$ cm and $I = 1.5$ A)

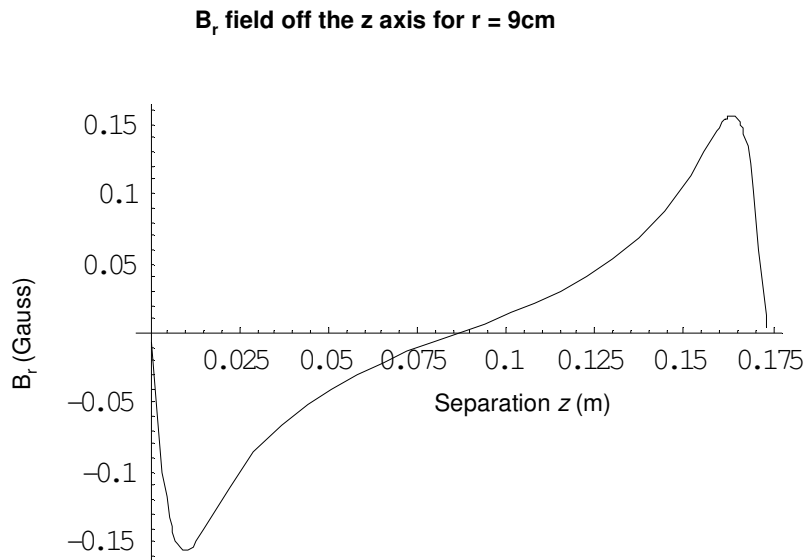


Figure 13 (field off the z axis for Maxwell pair coils such that $a = 10\text{ cm}$ and $I = 1.5\text{A}$)

Figure 14 and 15 show plots for the gradient G_z for different values of current and radius. In Figure 14 where the radius is twice as much as it was before we see decrease in the gradient value compare to Figure 9 by a factor of 3. However in Figure 15 the strength of gradient increases with increasing current, double current will double the value of the gradient (that make since B is proportional to I).

G_z field off the z axis for Maxwell pair coils

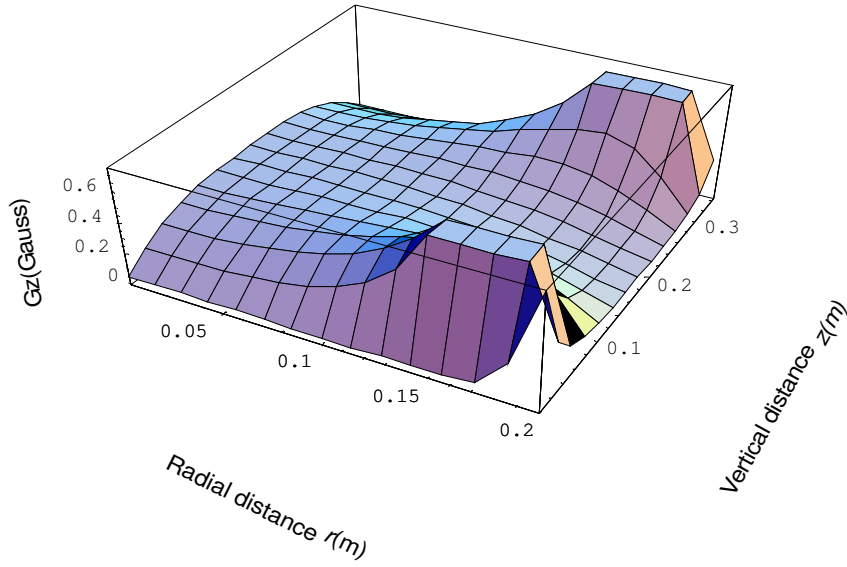


Figure 14 ($a = 20$ cm and $I = 1.5$ A)

G_z field off the z axis for Maxwell pair coils

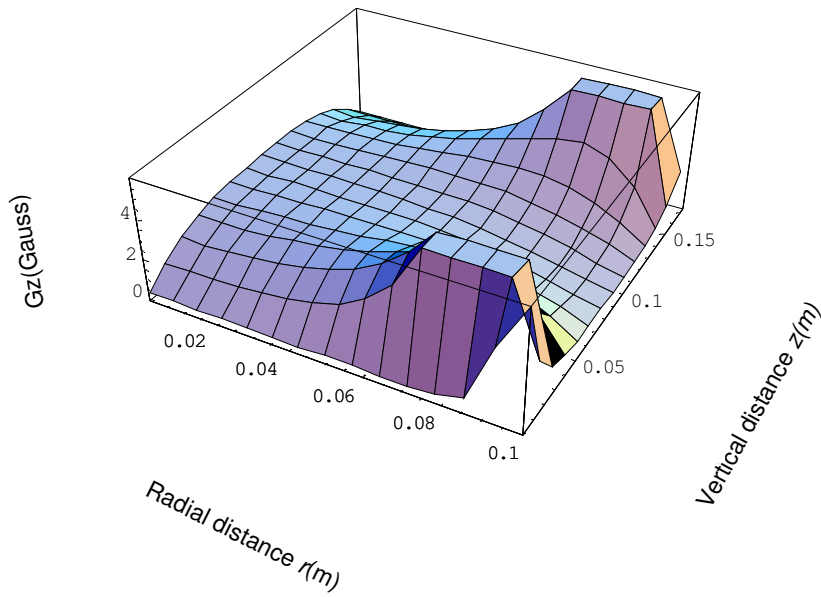


Figure 15 ($a = 10$ cm and $I = 3$ A)